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PRIME BI-IDEALS OF A NEAR ALGEBRA

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Abstract: The aim of this paper is to study the notion of a prime ideal, semi prime ideal and strongly prime ideal and their fuzzy concepts of a near-algebra. Some characterizations are obtained based on them.
Key words: Prime ideal, Semi prime ideal, Strongly prime ideal, Fuzzy set, Near-algebra
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1. INTRODUCTION

Generalization of ideals is necessary for further development in the study of algebraic systems. Yakabe [11] introduced the notion of quasi-ideals in near-rings. Lajos and Szasz [4] introduced the concept of quasi-ideals in associate near-rings. The quasi ideals are generalization of left and right ideals, where as bi-ideals are generalization of quasi ideals. We refer to Tamizh chelvam and Ganesan [10] for bi-ideals. Abbassi and Ambreen Zahara Rizivi [17] studied the prime ideals in near rings. The ideals in fuzzy sub near rings was introduced by Abou-Zaid [15].

A near-algebra is a near ring which admits a field as a right operator domain. Brown [1], Srinivas [8], Irish [3], Narasimha Swamy [5] have studied certain properties of near-algebra. In this paper, we introduce the concept of prime, semiprime, strongly prime bi-ideals of a near algebra, and obtain certain characterizations. It is also studied fuzzy concepts of prime, semi prime and strongly prime bi ideals for a near algebra.

2. PRELIMINARIES

Definition 2.1: An algebraic structure (N, +, .) is called a (right)near-ring if the following conditions are satisfied:

(i) (N, +) is a group (need not be abelian)
(ii) (N, .) is a semi group and
(iii) $(x + y)z = xz + yz$ for all $x, y, z \in N$

Definition 2.2: A sub set I of a near-ring N is said to be an ideal of N if: (i) (I,+) is a normal subgroup of (N,+), (ii) $IN \subseteq I$ and (iii) $n(n' + i) - nn' \in I \ \forall i \in I$, $n, n' \in N$.
If I satisfies (i) and (ii), then it is called right ideal of N.
If I satisfies (i) and (iii) then it is called left ideal of N.

Definition 2.3: A (right)near-algebra Y over a field X is a linear space Y over X on which multiplication is defined such that (i) Y forms a semi-group under multiplication, (ii) multiplication is right distributive over addition (i.e. $(a + b)c = ac + bc$ for every $a,b,c \in Y$) and (iii) $\lambda(ab) = (\lambda a)b$ for every $a,b \in Y$ and $\lambda \in X$.

Definition 2.4: A subset D of a near algebra Y over a field X is said to be a sub near algebra of Y if D is a linear subspace of Y and (D, .) is a sub-semi-group of Y.

Definition 2.5: A nonempty subset I of a near algebra Y is called a near algebra ideal of Y if:

(i) I is a linear subspace of the linear space Y,
(ii) $ix \in I$ for every $x \in Y$, $i \in I$,
(iii) $y(x + i) - yx \in I$ for every $x, y \in Y$, $i \in I$.

If I satisfies conditions (i) and (ii), then I is called a right ideal of Y.
If I satisfies conditions (i) and (iii), then I is called a left ideal of Y.

Definition 2.6: An element $d \in Y$ is called a distributive element of a near algebra Y if $d(a + b) = da + db$ for every $a,b \in Y$.

Definition 2.7: Let Y be a near-algebra over a field X. Then the set $Y_0 = \{a \in Y : a0 = 0\}$ is called zero-symmetric part of Y, and $Y_c = \{a \in Y / a0 = a\}$ is called constant part of Y. Further, Y is called zero-symmetric near-algebra if $Y = Y_0$, and Y is called constant near-algebra if $Y = Y_c$.

Notation 2.8: Let Y be a near-algebra and K and D be two subsets of Y. An operation * is defined by $K * D = \{a(a'+b) - aa' \in D \ / \ a, a' \in K, b \in D\}$.

Everywhere in this paper near-algebra means right near-algebra and Y is a zero-symmetric near-algebra.

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3. PRIME AND STRONGLY PRIME BI-IDEALS

In this section, we introduce the notion of prime bi-ideals and strongly prime bi-ideals of a near-algebra. We also study certain properties of these ideals.

Definition 3.1: An ideal P of a near algebra Y is called a prime ideal, if for any ideals D, K of Y, $DK \subseteq P \Longrightarrow D \subseteq P$ or $K \subseteq P$.
Definition 3.2: An ideal S of a near algebra Y is a semiprime ideal, if for any ideal D of Y, $D^2 \subseteq S \Longrightarrow D \subseteq S$.
Definition 3.3: An ideal P of a near algebra Y is a completely prime ideal, if for x, y $\in$ Y, $xy \in Y$ implies $x \in Y$ or $y \in Y$.
Definition 3.4: An ideal S of a near algebra Y is called a completely semiprime ideal of Y, if for any $x \in Y$, $x^2 \in S \Longrightarrow x \in S$.
Definition 3.5: A bi-ideal D of a near algebra Y is called a prime bi-ideal of Y, if $P_1P_2 \subseteq D \Longrightarrow P_1 \subseteq D$ or $P_2 \subseteq D$ for any bi-ideals P_1, $P_2 \in Y$.
Definition 3.6: A bi-ideal P of a near algebra Y is a semiprime bi-ideal of Y, if for any bi-ideal P_1 of Y, $P_1^2 \subseteq P \Longrightarrow P_1 \subseteq P$.
Definition 3.7: A bi-ideal P of a near algebra Y is called a strongly prime bi-ideal of Y, if $P_1P_2 \cap P_2P_1 \subseteq P \Longrightarrow P_1 \subseteq P$ or $P_2 \subseteq P$ for any bi-ideals of P_1, $P_2 \in Y$.
Theorem 3.8: Intersection of any family of prime ideals of a near algebra Y is semiprime ideal of Y.
Proof: Let $\{P_i: i \in I\}$ be a family of prime ideals of Y and let $P = \bigcap_{i\in I} P_i$. Consider an ideal S of Y such that $S^2 \subseteq P$. Which gives $S^2 \subseteq \bigcap_{i\in I} P_i$, implies $S^2 \subseteq Pi$ for all $i \in I$. Thus $S \subseteq P_i$ for all $i \in I$, as each P_i is a prime ideal of Y. So $S \subseteq \bigcap_{i\in I} P_i = P$. Hence $\bigcap_{i\in I} P_i$ is a semiprime bi-ideal of Y.
Theorem 3.9: Every prime bi-ideal of a near algebra Y is a semiprime bi-ideal of Y.
Proof: Let P be a prime bi-ideal of a near algebra Y, and P_1 be a bi-ideal of Y such that $P_1^2 \subseteq P$. This implies that $P_1P_1 \subseteq P \Longrightarrow P_1 \subseteq P$, since P is a prime bi-ideal of near algebra Y. Hence P is a semiprime bi-ideal of Y.
Proposition 3.10: If S is a semiprime ideal of a near algebra Y, then S is the intersection of all prime ideals containing S.
Theorem 3.11: Intersection of a family of prime bi-ideals of a near algebra Y is semiprime bi-ideal of Y.
Proof: Let $\{P_i: i \in I\}$ be a family of prime bi-ideals of near algebra Y. P_i′s are bi-ideals of Y and then intersection of family of bi-ideals is a bi-ideal of Y. Let P be a bi-ideal of Y such that $P^2 \subseteq \bigcap_{i\in I} P_i$. This implies $PP = P^2 \subseteq P_i$ for all $i \in I$. Thus $P \subseteq P_i$ for all $i \in I$. So $P \subseteq \bigcap_{i\in I} P_i$. Hence $\bigcap_{i\in I} P_i$ is a semiprime bi-ideal of Y.
Theorem 3.12: Every strongly prime bi-ideal of a near algebra Y is a prime bi-ideal of Y.
Proof: Let P be a strongly prime bi-ideal of near algebra Y. Let P_1, P_2 be two bi-ideals of Y such that $P_1P_2 \subseteq P$. Then $P_1P_2 \cap P_2P_1 \subseteq P_1P_2 \subseteq P$, implies $P_1P_2 \cap P_2P_1 \subseteq P$. Thus $P_1 \subseteq P$ or $P_2 \subseteq P$. Hence P is a prime bi-ideal of Y.
Proposition 3.13: A prime bi-ideal of a near algebra Y need not be a strongly prime bi-ideal of Y.
Example 3.14: Let X = {0,1} be a field with addition modulo 2 and multiplication modulo 2. Let Y={0,x,y}be a set with two binary operations + and . defined as:

+	0	x	y
0	0	x	y
x	x	0	y
y	y	y	0

.	0	x	y
0	0	0	0
x	0	x	x
y	0	y	y

Scalar multiplication on Y is defined by 0.x = 0,1.x = x for each $x \in Y$. The routine calculations show that Y is a near algebra over the field X. The bi-deals of Y are {0},{0, x},{0, y},{0,x,y}. Now {0} is a prime bi-ideal of N but not a strongly prime bi-ideal, as $\{0,x\}\{0,y\} \cap \{0,y\}\{0,x\} = \{0,x\} \cap \{0,y\} = \{0\} \subseteq \{0\}$. But neither {0, x} nor {0, y} contained in {0}. This shows that, every prime bi-ideal of Y is not a strongly prime bi-ideal of Y.

4. PRIME AND STRONGLY PRIME FUZZY BI-IDEALS

In this section we introduce the notion of prime fuzzy bi-ideals and strongly prime fuzzy bi-ideals of a near-algebra and obtained some fundamental results.
Definition 4.1: A fuzzy bi-ideal λ of a near algebra Y is called a prime fuzzy bi-ideal of Y if for any fuzzy bi-ideals γ, μ of Y, $\gamma o \mu \subseteq \lambda$ implies $\gamma \subseteq \lambda$ or $\mu \subseteq \lambda$.
Definition 4.2: A fuzzy bi-ideal λ of a near-algebra Y is called a strongly prime fuzzy bi-ideal of Y if for any fuzzy bi-ideals γ, μ of Y, $\gamma o \mu \cap \mu o \gamma \subseteq \lambda$ implies $\gamma \subseteq \lambda$ or $\mu \subseteq \lambda$.
Definition 4.3: A fuzzy bi-ideal λ of a near algebra Y is said to be a semiprime fuzzy bi-ideal of Y if for any fuzzy bi-ideals γ of Y, $\gamma o \gamma \subseteq \lambda$ implies $\gamma \subseteq \lambda$.
Definition 4.4: A fuzzy bi-ideal λ of a near algebra Y is said to be idempotent if $\lambda = \lambda o \lambda = \lambda^2$.
Theorem 4.5: The arbitrary intersection of prime fuzzy bi-ideals of a near algebra Y is a semiprime fuzzy bi-ideal of Y.
Proof: Let $\{\lambda i:i \in I\}$ be a collection of fuzzy bi-ideals of a near algebra Y. Then $\bigcap_{i\in I} \lambda_i$ is a fuzzy bi-ideal of Y. Let μ be a fuzzy bi-ideal of Y such that $\mu^2 \subseteq \bigcap_{i\in I} \lambda_i$. Then $\mu^2 \subseteq \lambda_i$ for all $i \in I$. Hence $\mu \subseteq \bigcap_{i\in I} \lambda_i$. Thus $\bigcap_{i\in I} \lambda_i$ is a fuzzy semi prime bi-ideal of Y.
Theorem 4.6: Every strongly prime fuzzy bi-ideal of a near algebra Y is a prime fuzzy bi-ideal of Y.

Proof: Let λ be a strongly prime fuzzy bi-ideal of a near algebra Y. Let any two fuzzy bi-ideals γ, μ of Y such that $\gamma o \mu \subseteq \lambda$. Then $\gamma o \mu \cap \mu o \gamma \subseteq \lambda$. Thus $\gamma \subseteq \lambda$ or $\mu \subseteq \lambda$.

Theorem 4.7: Every prime fuzzy bi-ideal of a near algebra Y is a semiprime fuzzy bi-ideal of Y.

Proof: Let λ be a prime fuzzy bi-ideal of a near algebra Y. For any fuzzy bi-ideal γ of Y be such that $\gamma o \gamma \subseteq \lambda$. Thus $\gamma \subseteq \lambda$. Hence λ is a semiprime fuzzy bi-ideal of Y.

Remark 4.8: Every fuzzy bi-ideal of Y is semiprime, but every fuzzy bi-ideal of Y is not prime.

Proof: Consider fuzzy bi-ideals γ, μ and λ of near algebra Y={0,1,2} given by $\gamma(0) = 0.7$, $\gamma(1) = 0.6$, $\gamma(2) = 0.4$, $\mu(0) = 1$, $\mu(1) = 0.5$, $\mu(2) = 0.3$, $\lambda(0) = 0.7$, $\lambda(1) = 0.65$, $\lambda(2) = 0.3$. Then $(\mu o \lambda)(0) = 0.7$, $(\mu o \lambda)(1) = 0.5$ $(\mu o \lambda)(0) = 0.3$, where $\mu o \lambda \leq \gamma$, but neither $\mu \leq \gamma$ nor $\lambda \leq \gamma$. Hence γ is not a prime fuzzy bi-ideal of Y.

5. REFERENCES:

[1] H. Brown, Near-algebra, Illinois J. Math. 12, 215-227 (1968).

[2] R.A. Good, and D.R. Hughes, Bull. Am. Math. Soc.58, 624-625 (1952).

[3] J.W. Irish, Normed near-algebras and finite dimensional near-algebras of continuous functions, Doctoral thesis, University of New Hampshire (1975).

[4] S. Lajos and F. Szasz, Acta. Sci. Math. 32, 185-193 (1971).

[5] P. Narasimha swamy, A Note on fuzzy near-algebras, International Journal of Algebra, 5(22), 1085-1098 (2011).

[6] G. Pilz, Near-rings, North Holland, Amsterdam (1983).

[7] B. Satyanarayana, Contribution to near-ring theory, Doctoral Dissertation, Acharya Nagarjuna University (1984).

[8] T. Srinivas, Near-rings and application to function spaces, Doctoral Dissertation, Kakatiya University (1996).

[9] O. Steinfeld, On ideal-quotients and prime ideals, Acta Math. Acad. Sci. Hung. 4, 289-298 (1953).

[10] T. Tamizh Chelvam and N. Ganesan, On bi-ideals of near ring, Indian J. Pure Appl. Math., 18 (11):1002-1005 (November 1987).

[11] I. Yakabe, Quasi-ideals in near-rings, Math. Rep., XIV, 41-46 (1983).

[12] M. Shabir, Y.B. Jun and M. Bano, On prime fuzzy bi-ideals of Semigroups, Iranian J. Fuzzy Systems, Volume 7, Number 3, 115-128 (2010).

[13] Shahid Bashir, Prime bi-ideals and strongly prime fuzzy bi-ideals in near rings, Annals of Fuzzy Mathematics and Information, Volume 9, Number 1, 125-140 (January 2015).

[14] K. Syam Prasad and B. Satyanarayana, Fuzzy prime ideals of Gamma-near-rings Soochow J. Mathematics Volume 31, Number 1, 121-129 (January 2005).

[15] Salah Abou-zaid, On fuzzy sub near-rings and ideals, Fuzzy Sets and Systems, 44, 139-146 (1991).

[16] Y.B. Jun and H.S. Kim, A characterization theorem for fuzzy prime ideals in near-rings, Soochow J Math., 28:1, 93-99 (2002).

[17] S.J. Abbassi and Ambreen Zahra Rizvi, Study of prime ideals in near-rings, Journal of Engineering and Sciences, Volume 2, Number 1 (2008).

[18] P. Narsimha Swamy, B. Jyothi, Rakshita Deshumukh and T. Srinivas, Quasi-ideals and Bi-ideals of Near-Algebra, International Journal of Engineering, Science and Mathematics, Volume 7, Issue 3, 382-386 (March 2018).